ONE-ASSET HANK MODEL

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This is a quick explanatory documentation showing the one-asset HANK model being solved in the given set of files. If you got the document separately from the codes, codes are available at `https://github.com/gregkaplan/phact/tree/master/examples/one_asset_HANK`. For an example run of the program, go to `https://sehyoun.com/EXAMPLE_one_asset_HANK_web.html`.

1. Household

Households maximize their utility

$$\max_{c,\ell} \int_0^\infty e^{-\rho t} \left(\frac{c^{1-\gamma}}{1-\gamma} - \phi_0 \frac{\ell^{1+\frac{1}{\phi_1}}}{1+\frac{1}{\phi_1}} \right) \mathrm{d}t$$

Households are heterogeneous in two dimensions: their income state and their (liquid) asset position. Income (z) is an exogenous stochastic process. Assets evolve according to

$$\mathrm{d}a = (r \cdot a + (1-\tau) \cdot w \cdot z \cdot \ell + T + \Pi - c)\,\mathrm{d}t$$

where

τ	tax
r	interest rate
w	wage
z	labor productivity
ℓ	labor supply
T	lump-sum (governmental) transfer
Π	profit share
c	consumption

with borrowing constraint:

$$a_t \geq \underline{a}.$$

For simplicity, income is assumed to have two states and to follow a Poisson process with intensity $\lambda(z)$. Given this model setup, the corresponding Hamilton-Jacobi-Bellman equation is

$$\begin{aligned}\rho V(a,z) = \max_{c,\ell} u(c,\ell) &+ (r \cdot a + (1-\tau) \cdot w \cdot z \cdot \ell + T + \Pi - c)\,\partial_a V(a,z) \\ &+ \lambda(z)(V(a,z') - V(a,z))\end{aligned}$$

Date: 2017/10/25.

1.1. **Profits Π.** In the two-asset model of Kaplan, Moll and Violante (2017), there is a natural way of distributing firm profits. One would include the profit share as part of the return of the illiquid asset. However, the natural counterpart does not exists for one-asset model. In this one-asset example, profits are transferred to households proportional to their income level. This assumption is meant to minimize the redistribution implied by cyclical fluctuations in profits.

2. Production

2.1. **Final Good Aggregator.** There is a representative final good producer, which produces the final good using the CES aggregator.

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

2.2. **Intermediate Good Producers.** Intermediate good producer firms use only labor

$$y_{j,t} = n_{j,t}$$

Firms have price adjustment costs given by

$$\Theta_t \left(\frac{\dot{p}_t}{p_t} \right) = \frac{\theta}{2} \left(\frac{\dot{p}_t}{p_t} \right)^2 Y_t$$

where Y_t is the aggregate output. This generates the continuous time version of the familiar New Keynesian Phillips Curve: [1]

$$\left(r_t - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\epsilon}{\theta}(m_t - m^*) + \dot{\pi}_t$$
$$m^* = \frac{\epsilon - 1}{\epsilon}$$

The linearized version of this equation does not feature Y_t because $\dot{Y}_t$ is zero in steady state. Therefore, in the codes, we directly write this equation as

$$r_t \cdot \pi_t = \frac{\epsilon}{\theta}(m_t - m^*) + \dot{\pi}_t$$

noting that they are equivalent under linearization instead of introducing an extra variable. Also, note that π_t is a "choice"/jump variable.

3. Monetary Policy

Monetary Policy is set using the following Taylor rule.

$$i_t = \overline{r_t} + \phi_\pi \cdot \pi_t + \phi_y (y - \overline{y}) + \varepsilon_{MP,t}$$
$$\mathrm{d}\varepsilon_t = -\theta_{MP}\varepsilon_t + \sigma_t \cdot dW_t$$

From Fisher equation, we have

$$r_t = i_t - \pi_t$$

[1] Refer to Kaplan et al (2017) equation (19) for more details.

4. Government

Government satisfies the budget constraint given by

$$\dot{B}_t^g + G_t + T_t = \tau_t \int w_t z \ell_t(a,z) g_t(a,z) \mathrm{d}a\mathrm{d}z + r_t B_t^g$$

For this example, government will be assumed to satisfy budget constraint by adjustment transfers, T_t.

Also, for added flexibility, B_t^g is allowed to have the form

$$\dot{B}_t^g = \phi \pi B_t^g$$

where $B_t^g = B^g_{\text{steady state}}$ if $\phi = 0$.

Plugging this functional form into the government budget constraint gives

$$T_t = \tau_t \int w_t z \ell_t g_t(a,z) \mathrm{d}a\mathrm{d}z + r_t B_t^g - G_t - \phi \pi B_t^g$$

5. Equilibrium

Every market is standard except for the bond market. The government is the only issuer of debt, so

$$B_t^g = \int a g_t(a,z) \mathrm{d}a\mathrm{d}z$$

The labor market clearing condition is

$$\text{(labor supply)} \quad \int z \ell_t(a,z) g_t(a,z) \mathrm{d}a\mathrm{d}z = L_t \quad \text{(labor demand)}$$

The goods market clearing condition is then implied by Walras' Law.